A STUDY OF EXPERIMENTAL WORK-TIME DISTRIBUTION CHARACTERISTICS TO DETERMINE THE EXISTENCE OF A TYPICAL DISTRIBUTION

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By

Paul Harold Friedman

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A STUDY OF EXPERIMENTAL WORK-TIME

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ABSTRACT

The characteristics of an operator's work pattern must be known before time study can be placed on a theoretically sound scientific foundation. Therefore the objective of this thesis is to study the characteristics of experimental work-time frequency distributions and to determine the existence of a typical distribution for statistically stable operators on a short, manually controlled repetitive operation. The data are obtained from two studies of a research project begun at the Georgia Institute of Technology under the direction of Doctors Lehrer and Moder in 1951.

The experimental distributions are tested for statistical stability. Only stable distributions are used in further analytical work which include the calculation of the mean, variance, skewness, and peakedness of each distribution as well as the fitting of the Normal, Log Normal, and Pearson Type III curves to each distribution.

In view of the experimental situation, it seems reasonable to conclude that there is a theoretical work-time distribution which typifies work-time phenomena for this type of activity. This theoretical work-time distribution appears to have the following characteristics:

1. It differs significantly from the Normal Curve.
2. It is positively skewed.
3. Its peakedness is greater than that of the Normal Curve.
4. It can be reasonably approximated by a Pearson Type III Curve.

The various curve functions that were fitted to the thesis data indicate that the Normal Curve and the Poisson Curve do not adequately fit the data, that the Log Normal Curve does fit part of the data, and that the Pearson Type III Curve fits a substantial portion of the data.

These conclusions should be viewed in light of the limitations of the present investigation. The sample sizes range for individual work-time distributions from 125 to 150 in Study A and from 70 to 92 in Study B. Because of the criteria of statistical stability, the number of distributions analyzed is limited to six from Study A and seven from Study B. A further limitation on the conclusions is that the operation is manually controlled and the cycle times are relatively short. Also, the operators seem to be highly motivated. In Study A, the shortness of the overall cycle time in conjunction with the stop watch limitations prevent the elimination of all foreign elements to the standard method. The foreign elements are more readily eliminated with the micromotion study since the film can be viewed repeatedly and at a slower rate than the original operation.

It is recommended that another film be taken of the same operation in which a large sample size is recorded for each operator in as short a time interval as possible in order to minimize variation of operator pace during the observation period. It is further recommended that the distributions from the proposed film be tested for stability. If found to be stable, the distributions should be fitted to the Normal, Log Normal, Pearson Type III, and other suitable curves to test the hypothesis formulated at the conclusion of this thesis that the work-time distributions of statistically stable operators can be typified by the Pearson Type III Curve.

CHAPTER I

INTRODUCTION

Work-Time Frequency Distributions.--Since the human being is not an automaton, he cannot repeatedly perform a given task in identical time intervals. These time intervals may vary according to some predictable pattern, or they may vary in an erratic manner. If times obtained from a given task are plotted on a graph with the occurrence frequency of each time interval as the ordinate and the time scale as the abscissa, a work-time frequency distribution will result. This distribution, upon the application of statistical techniques, will reveal the pattern of performance if one exists. The discovery of the existence of a typical work pattern would be helpful to the theory of industrial psychology as well as the theory of industrial engineering.

Research on this subject was begun in the School of Industrial Engineering at the Georgia Institute of Technology under the direction of Doctors Lehrer and Moder in 1951. The objectives of the study have been to analyze statistically the worker's performance at a given task in order to gather basic information about the work-time performance pattern.

The Operation Studied.--The operation selected for this project was the manual assembling of a ball point pen. The parts for each assembly were 1. barrel and clip sub-assembly, 2. writing unit, 3. drive nut, and 4. ferrule. The operation consisted of the following elements:

1. Get barrel
2. Place barrel in fixture
3. Get writing unit
4. Place writing unit in barrel
5. Get drive nut
6. Place drive nut on barrel
7. Get ferrule
8. Place ferrule over drive nut
9. Remove complete unit from fixture
10. Place complete unit in staking machine
11. Stake ferrule to secure assembly
12. Aside assembled unit to box

Each hand performed identical and simultaneous motions. Therefore, two units were assembled during each cycle.

Although this operation had been standardized, some of the nineteen operators studied varied in the placement of the barrels and the asiding of the completed assemblies. However, each operator maintained a constant, though different, method. The operators did not work under a financial wage incentive, but they did have a standard or goal that the company expected them to reach each day. This provided an incentive since failure to reach this pre-determined goal could possibly have resulted in transfer or lay-off. The operators seemed highly motivated and regarded their job as one involving prestige.

<u>Previous Studies</u>.--In connection with this project, Lind (1) investigated variations in the raw, unadjusted work times through a stop watch study.

Because of the shortness of the overall cycle time and the limitations of a stop watch, Lind recommended that an extensive micromotion study be made of the same operation in order that variations from the standard method might be more easily detected. Taft (2) performed this micromotion study by taking eight film shots of each operator at one hour intervals. Because of mechanical failure in the drive mechanism of the camera, some of the shots were discarded. The remaining film was analyzed frame by frame. Cycles showing defective parts, changes in method, the asiding of a handful of parts, or the obtaining of a new supply of parts were removed from the study. This left an average of ten usable cycles for each film shot.

Since most of the method variations occurred in the last element, the total cycle time was reduced by the time involved in this last element to make the data more consistent and was referred to by Taft as the modified cycle time. It was impossible to remove the time for the last element in Lind's data due to the manner in which the data were obtained.

In the theses by Lind (1) and Taft (2) based on these two studies, it was concluded that the work-time frequency distributions of all the operators studied were positively skewed. This lead to the general hypothesis that there is a distribution pattern which will typify the distributions of all statistically stable operators performing short cycled manual operations. The above stated hypothesis will be tested for the specific operation used in the two previous studies, and the data for this thesis will be taken from these two studies.

CHAPTER II

LITERATURE SURVEY

Criticism of Current Practice

Many people in the field are of the opinion that before time study can be classified as even approaching a science, there must be only one method used and that method must be theoretically sound, reliable, and valid. There are several methods for conducting a time study in use today. The advocates of almost all of the methods each claim accurate scientific results. With each of the large number of methods in use yielding different results, all of them cannot be valid. Davidson asserts,

> There are such significant differences among the three systems [Work Factor, Methods Time Measurement, and Holmes' Method] that if any one of them might be accurate, the other two cannot be....All are equally suspect, and until the advocates of some one of them can produce objective evidence of accuracy we prefer to regard all the claims with some skepticism. (3)

One of the greatest reasons for time study being unscientific is the subjective rating or normalizing of the observed data. Much is being done to indoctrinate time study men into using the same performance standards both within a company and within an industry. This will promote consistency, but it will not remove the original subjectivity in determining the standard. Mundel (4) takes a step forward in decreasing subjectivity in rating by dividing the rating process into first comparing the performance with a standard rate of activity and then comparing the job difficulty with that of the agreed standard. This certainly decreases the degree of subjectivity, but subjectivity is still present in establishing the original standard and in applying the two steps in the rating technique. The theoretical foundation for this procedure, as for all other known procedures is also questionable since the phenomena of work-time variations are not adequately understood.

Another questionable practice in use today is the arbitrary discarding of extreme values found in the observations. Presgrave states,

> Any recording that is beyond the normal range cannot be ignored with safety....Unusually low timings...may provide the key to the whole study. Unusually high timings are often caused by fumbles, but also may be an indication of conditions that must be corrected or taken into the standard rate. (5)

A common error is the relying on past or present production figures for determining what the standard should be. Presgrave condemns this practice by stating, "...output is not necessarily a true measure of speed and that an operation done in two different ways provides, for scientific purposes, a situation that is no different from that created by two entirely different operations." (6)

The use of standard data will reduce the errors found in other methods, but it introduces another error which might be of equal consequence. This error relates to the implied additiveness of elements. Certain elements by their inherent nature have, in varying degrees, an interaction with the preceding and following elements. Thus, there are many elements which are not additive.

Work Performance

Workers on the same job differ in their work methods for various reasons. Some of these are psychological and physiological differences, utilizing their own developed secret method, and differences in training. Even difference of pace may affect the work method. Sometimes it is easier to use one method utilizing fewer restraining muscles while working slowly, and to use an entirely different method utilizing more restraining muscles while working rapidly. Presgrave has made a good analogy to this situation in comparing a baseball player to an industrial worker by stating that a ball player,

> Running leisurely...can follow the diamond-shaped course exactly. Running at top speed he is forced to follow a curved course or else lose time by plus and minus accelerations at the bases. Speed necessitates a change in the runner's method. In fact the faster he runs the farther he runs.... (7)

<u>Influences on Work-Times</u>.--Work times are subject to many outside influences which affect the times even more than random chance factors. These influences may either aid or hinder by lowering or raising the work-times. Wiberg states, "Examples of aids are spurts, learning a better method, incitement (warming-up effect), a sharper or better tool, etc. Examples of hindrances are fatigue, boredom, illness, annoyances, distractions, a duller or poorer tool, material changes, etc." (8) These aids and hindrances, or assignable causes as they are often called, should be eliminated as much as possible from a time study unless the aids can be fully and consistently utilized and unless the hindrances are inherent in the work.

Restriction of output is another vital influence on work times. Wiberg states, "Such misrepresentation generally occurs either as a variation from the standard method, or as a general slowing down of the movements, or as a combination of variation and slowing down." (9) On the other hand to explain the decrement phenomenon, Davis and Josselyn hypothesize, "The operator uses the same work method and continues to work at the same rate of speed whenever the operation is performed, but introduces more and longer stoppages...." (10) This hypothesis was formulated when their study of a repetitive assembly operation revealed that the effective time for performing each element did not vary significantly throughout the hours of the day, but that the production varied significantly throughout the day in the form of a work decrement pattern. They further state that productivity may be increased by decreasing personal delays and not solely by decreasing the effective operation time.

Factors Influencing Pace.--Mundel describes the factors influencing the work pace when he states,

> The actual pace of performance observed [assuming external factors do not change] must be understood to be a function of the skill, familiarity with work, aptitude for the work and exertion at the work of the operator, but these variables are hardly separably identifiable.
>
> Skill is taken to mean proficiency at following a given muscular pattern. Pace is taken to mean rate of muscular activity....The operator's aptitude determines how fast a pace he can maintain, or how long a period it takes him to acquire the skill required for a rapid pace.... Familiarity refers to the exposure of the operator to opportunity to utilize his aptitude to acquire skill and affects maximum potential pace. (11)

It might be important to recognize that motivation is omitted by Mundel as an influencing factor on the pace. Motivation will influence, to a certain extent, the exertion of an operator at a given job.

Wiberg suggests three factors as influencing pace: motivation, habituation, and aptitude. (12) As there should be, there are overlappings of some of the factors mentioned by Mundel and Wiberg. All of these factors and possibly many others certainly have an effect on the performance pace.

Need for Research

Fallacies in Previous Research.--Much research has been undertaken to improve the reliability of time study. Some of the researchers have made definite contributions to this purpose, but others have allowed their imaginations to lead them to the drawing of unfounded conclusions based on insufficient or unreliable data. Davidson states, "...the arbitrary imposition of a rule on a set of experimental data has been one of the most unfortunate improvisations in the development of time study." (13)

Davidson further says, "...the collection and manipulation of data according to some superficially presumed relationship is an inadequate stimulation of the scientific method." (14) Presgrave enforces this by saying,

> ...there is always the danger of drawing analogies, which must not be carried too far, no matter how interesting the speculation to which they give rise. At the same time there are undoubtedly certain basic rules which, recognized and applied with discretion, will go far in organizing our thinking and in defining and directing procedure. (15)

Another fallacy occurring in some of the previous studies is the basing of research on past production records or on previous time studies which were not made for research purposes. Since the researcher knows little about surrounding circumstances or other external influences which might have been present when the past production was being performed or when the time studies were performed, these figures are of little use to him. Time studies for research purposes require a fine degree of accuracy and a close observation of variations in method, as well as an account of experimental conditions present at the time of the study.

Exploring New Areas.--The elimination of subjectivity must be accomplished if time study is ever to be reliable. So far, no one has been able to measure a performance objectively or to objectively determine what normality is. This is virgin territory as far as research is concerned because of the formidable problem and the experimental difficulty that would be incurred.

Buffa suggests that a national performance standard should be developed. "Such a standard would do for performance rating exactly what the platinum-iridium bar has done for linear measurement." (16) The Society for the Advancement of Management has been conducting a project along these lines by setting up a series of motion picture films depicting the normal pace in several areas. This standard might show a high industrial practicality in supplying consistent data even though the subjective judgment used in establishing the standard might be inaccurate.

Performance-time phenomena, a comparatively new area of research, is being studied at the Ohio State University. It is hoped that this study "...may lead to the development of new concepts and operationally valid methods." (17) Research along similar lines was initiated in 1951 at the Georgia Institute of Technology under the direction of Lehrer and Moder. In this project, Lind (1) and Taft (2), under separate studies of the same operation, statistically analyzed work-time distributions obtained from stop watch studies and micromotion studies respectively.

Compiling Statistical Information.—The abstract nature of translating data into a practical standard has been a sore spot for time study practitioners, "...whereas his fellow scientific research workers have turned to statistics." (18) Wiberg infers that if time study is to be scientific, it must be analyzed by using statistics and industrial psychology. The variations occurring in repeated performances should be analyzed to determine the causes. This can most easily be done from a statistical approach. (19) To further emphasize the need for statistical information, Presgrave states that the necessary estimating of operator speed exists because of a lack of statistical information rather than the lack of scientific instruments. (20) With the far sighted outlook of these men in mind, it is apparent that more statistical information of the worker's performance times must be gathered and analyzed in the form of research if we are to advance time study methodology.

Statistics as an Aid

Determining Parent Populations.—It is impossible to determine what the exact parent population of a performance time should be, but statistics can give us a practical and reliable estimate of this population. Presgrave states, "If it were possible to obtain a broad enough sampling of any operation, there would be no need for anything but the routine recording of time." (21) Thus, if we can obtain an insight of the parent population and determine the distribution of the sampling statistic within that population, the problem of normalizing will be resolved.

Wiberg infers a parent population for particular types of job performances in developing graphs and distributions for these types of performances. (22) Gomberg states the opinion that Wiberg's graphs and distributions have their principal value in the fact that, "...this is the first time an effort has been made to apply the methods of statistical inference to time study analysis on an inferential basis rather than merely as a descriptive attempt to summarize data." (23)

<u>Fluctuations in a Sample</u>.--Fluctuations in a sample will occur, if for no other reason, because of inherent randomness. Gomberg states, "Since we have defined a state of statistical control as a balanced constant chance cause system, we can always expect fluctuations in sample time studies. However, these fluctuations should be predictable from probability theory." (24) A sample may also fluctuate because of variations which are due to reasons other than randomness. Needless to say, these variations which arise from assignable causes should be eliminated or at least reduced as much as possible.

<u>Utilizing Control Chart Procedures</u>.--The control chart is used to determine the stability of a process or an operation and to guide the procedure of elimination of assignable causes of variations. In defining a control chart as it is applied to quality control, Brownlee states, "The statistical quality control chart is a simple means of distinguishing between

that part of the variability which is probably inherent in the process, and that part which is due to abnormal discrete events." (25) Statistical quality control procedures can be applied equally well to work measurements. Lehrer points out that statistical work measurement control,

> ...makes possible the detection of extreme variations in rate of production or productivity, and allows concentration of effort on correction of conditions causing these extreme variations. Progressive elimination of assignable causes will allow better utilization of industrial facilities and eventually a stable pattern of variation indicative of the presence of a true 'system of chance causes' will become evident. (26)

The control chart is applicable regardless of the distribution of the underlying population because, except for very small samples, sample means have an approximately normal distribution. (27)

Need for Stability.—Unless the observed data from an operation is statistically stable, very little can be gained from this data. Any conclusions or inferences drawn from statistically unstable data must be viewed with skepticism. This does not mean that such inference are in error, but that they are inconclusive.

The variations which require stabilization are those arising from mechanical, physiological and sociological sources. Gomberg warns that unless the variations from these sources are stabilized, "It will be difficult, if at all possible, to predict any meaningful fixed production standard from a time study." (28)

Statistically stable data is often difficult to obtain through a stop watch time study. In an analysis of Lind's (1) study, it was found that the performance of 14 out of the 19 operators studied were unstable and only one operator's performance could be considered completely stable. For this reason, Lind recommended that a micromotion analysis be made on the same operators used in his study. The film analysis could reveal variations that would be undetectable during the course of a stop watch time study and thus should provide more stable data when these assignable variations are removed.

Distribution Characteristics

Work-Time Distributions as a Tool.—Wiberg defines a work-time distribution in stating,

> A work-time distribution is defined as a frequency distribution of a specified number of time values obtained through time study as actual and unadjusted watch readings, on an element of repetitive manual work, for the purpose of relating the distribution characteristics to the various influences which the worker, the work-method, and the work-environment exert upon the efficiency of a particular work situation. (29)

He feels that the work-time distribution "...would replace time study ratings (leveling) where they seem too subjective, even when guided or checked." (30) The work-time distribution, when fully developed, can reveal "...how well any manual worker is selected, trained, and motivated for the work." (31) The work-time distribution is an excellent tool, but it appears that there is no evidence that all of the aforementioned information can be obtained from it.

Inferences Drawn from Distributions.—Wiberg compares the type of obtained distribution with a human trait. He states,

> When work-times are arranged into frequency distributions...analysis relates the distribution characteristics to the following worker influences, (sic) Skew is related to motivation, width is related to habituation, and minimum time is related to aptitude. The stronger the motivation and habits, and the more suitable the aptitudes of the worker are for the work, the greater is the skew, the smaller is the width, and the lower is the minimum time respectively, for the distribution. (32)

Wiberg infers that restriction of output can be detected with the aid of a work-time distribution. When a worker secretively misleads a time study observer by working at a methodically slower pace, his "...distribution tends to lack skew and to show a range smaller than is ideally possible, such as where the maximum is less than 50 per cent greater than the minimum time." (33) This is so because a person can approximate the same times at a slower pace than at a faster one; however, this is apt to confuse restriction of output with habituation in Wiberg's distributions.

Sometimes the inferences drawn from distributions can appear to be contradictory because of the level of significance used in the statistical tests. Rothe illustrates this occurrence in his study of butterwrappers. He states,

> All of the distributions were tested for normality according to the Chi Squared test....At the one per cent level of significance, none of the individual distributions differed significantly from normal....Using the method of beta coefficients to test for skewness and kurtosis, the distributions for operators 1, 2, and 7 were significantly leptokurtic. This apparent discrepancy between the Chi Square and the beta coefficient tests for normality may be attributed to the use of the one per cent level in the Chi Squared Test,—a procedure that tended to make it difficult for the distribution <u>not</u> to be normal. (34)

Another reason for the discrepancy Rothe found is that various statistical methods used to test normality do not necessarily have the same statistical efficiency.

Skewness as a Measure of Performance.--Skewness occurs in work-time distributions because at one end of the curve there is a physiological limit which may be often approached, but at the other end of the curve there is no time limit. Very little can be done to shorten the time, but most variations to the standard method tend to lengthen the time, thus causing skewness. Davidson states that the probability of a worker leaving a certain type of work would be inversely proportional to his aptitude and that natural selection would cause the curves to tend toward skewness. (35) Bedford in his shoe factory study found that distributions of the fast workers were positively skewed while those of the slow workers were negatively skewed. (36) On the other hand, Rothe found in his studies that the fast workers were not positively skewed nor were the slow workers negatively skewed. It must be remembered, however, that Rothe stated, "There were so many slight variations in techniques among the operators that a detailed motion analysis was infeasible for the present purpose." (37) With variations in method, any inferences drawn are inconclusive. Lind (1) and Taft (2) in their respective studies found that practically all of the workers' distributions were positively skewed. This seems to enhance Wiberg's suggestion that a skewed distribution indicates motivation as it is the writer's opinion that the workers used in Lind's and Taft's studies were highly motivated.

Presgrave is of the opinion that the skewness which shows up is not always inherent. He says, "In practically every case it is possible to visualize, or even detect, the existence of some secondary characteristic or of some inhibiting factor either of which would distort the curve." (38) This statement recalls the need for stability before accurate conclusions may be drawn. If a stable operation indicates skewness, there is a very great likelihood that the skewness is inherent since stability infers that variations in performance times are due only to random chance causes, and that there are no assignable causes for variations in performance present.

Conclusion

The lack of knowledge concerning the performance-time pattern indicates that basic research should be conducted to discern as much as possible about this phenomenon. Some information has already been gathered, but there is a great deal more to be learned. In the writer's opinion, there is sufficient experimental evidence to conclude that the work-time distributions of statistically stable operators are positively skewed. With this established, the next area for research should be to find out quantitatively how skewed the distributions are and what effect the efficiency of the worker has on the amount of skew. In further search for the characteristics of the work-time pattern, the degree of peakedness should also be ascertained quantitatively. Next, the existence of a typical curve which would statistically represent the performance-time distribution should be determined. For the conclusions drawn from such a study to be meaningful, the data should be drawn from a large sample and should be taken from statistically stable distributions.

CHAPTER III

OBJECTIVE

Since the phenomena of work-time patterns are not adequately understood, it would be helpful to obtain as much information as possible concerning them. Certain portions of the work-time pattern are studied in this thesis. An investigation is made of the characteristics of an operator's work pattern for a specific operation which is short, repetitive, and manually controlled. The objective of this thesis is:

(1) To test the hyposthesis that there is a work-time distribution pattern which will typify the statistically stable distributions of operators performing short cycled manually controlled operations.

(2) To determine the moments of statistically stable work-time distributions from operators on a short, manually controlled repetitive operation.

CHAPTER IV

PROCEDURE

Data for this thesis was taken from the studies conducted by Lind (1) and Taft (2) of the manual assembly of a ball point pen. Since there was a changing of operators from one shift to another and a slight labor turnover during the period between the two studies, the numerical designation of operators in Taft's study did not conform to that in Lind's study. The numerical designations of the two studies were made to conform by introducing a coded system consisting of four characters. The first character, A or B, represented Study A or Study B respectively, and indicated from which study the particular distribution came. The second character, 1, 2, or 3, corresponded to the shift in which the operator was working at the time the study was made. The third character, from 1 to 19, referred to the operator number assigned by Lind or Taft in their respective studies. The fourth character was an alphabetical letter which was assigned to an individual operator. When an operator appeared in both studies, the fourth character in the numerical designation of the operator was the same in both studies. This coding appears as Table 1.

Stability.—The stability of the distributions in the stop watch study, hereafter called Study A, was determined by interpretation of the control charts developed by Lind. The criteria for instability as used in this thesis were 1. points out of the three-sigma control limits, 2. significant

Table 1. Numerical Comparison of Operators in Study B with Study A

STUDY A		STUDY B	
Opr. No. Assigned By Lind	Coded	Opr. No. Assigned By Taft	Coded
1	A1-1A	- -	
2	A1-2B	4	B1-4B
3	A1-3C	- -	
4	A1-4D	1	B1-1D
5	A1-5E	2	B1-2E
6	A1-6F	5	B1-5F
7	A1-7G	7	B1-7G
8	A2-8H	8	B2-8H
9	A2-9J	13	B2-13J
10	A2-10K	9	B2-9K
11	A2-11L	- -	
12	A2-12M	10	B2-10M
13	A3-13N	19	B3-19N
14	A3-14P	16	B3-16P
15	A3-15Q	18	B3-18Q
16	A3-16R	17	B3-17R
17	A3-17S	14	B3-14S
18	A3-18T	15	B3-15T
19	A3-19U	- -	
- -		3	B1-3V
- -		6	B1-6W
- -		11	B2-11X
- -		12	B2-12Z

runs of eight or more consecutive points on one side of the mean, or
3. trends during the period studied.

The analysis of variance was used to determine the stability of the distributions in the micromotion study, hereafter called Study B, by comparing the variance of the time values within each shot to the variance of the shot averages. As shown in Table 12 in the Appendix, a tabulation was made of the sum of the individual cycle times per shot as well as the sum of the squares of the individual cycle times per shot.

The summation of the individual cycle times for each shot was squared and designated as "T_i" in further analytical work. The other symbols used in the analysis of variance were:

T	Sum of cycle times for an operator
n_i	Number of cycles in a shot
N	Total number of cycles for an operator
X_{ij}	Individual cycle where "i" designates the film shot and "j" designates the cycle within the shot

Sample calculations for operator B2-10M, as well as formulae used in the computations, are shown in Figure 14 in the Appendix. Only the data from distributions which exhibited stability were used in further analytical procedures. The raw times, in frames, were used in the analysis of variance, but the raw times were coded into intervals of ten or multiples of ten frames for the remainder of the calculations to make the data less voluminous.

<u>Moments</u>.—The work times of the stable operators, as individuals and as a group, were tabulated according to frequency of cycle times occurring

within time intervals.[1] These tabulations appear as Tables 10 and 11 in the Appendix. From these tabulations, the mean, variance, skewness, and peakedness for each distribution were calculated according to the procedure shown in Figure 15 in the Appendix. The statistic "a", mean deviation divided by standard deviation, was used as a measure of peakedness since the sample size was less than 200. (39)

Skewness and peakedness were then checked for normality by use of confidence level tables. (39)

Curve-Fitting.—The experimental work time distributions of stable operators, as individuals and as a group (See Tables 10 and 11 in the Appendix), were fitted to a Normal curve having the formula

$$Y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$$

and tested for goodness of fit by use of the Chi-Squared test. Sample calculations are shown in Figure 16 in the Appendix. The probability level of the fit was determined by the use of a nomograph. (40)

The experimental work-time distributions of stable operators, as individuals and as a group, were fitted to a Log Normal Curve by putting the time intervals in terms of logarithms and using "t" values to enter normality tables. (41) They were tested for goodness of fit by use of the Chi-Squared test. Sample calculations are shown in Figure 17 in the Appendix. The probability level of the fit was then determined by the use of a nomograph. (40)

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[1] For convenience in the calculations throughout this thesis, all times taken from Study A are expressed in hundredths of a minute.

The experimental work time distributions of stable operators, as individuals and as a group, were tested for a Poisson Curve which has two of its parameters, mean and variance, equal. Since the distributions of all of the operators had these two parameters significantly different, no further tests were made to fit the data to a Poisson Curve.

Kenney (42) states that if a Pearson Type III curve is to represent an experimental distribution, the moments of that distribution should satisfy, at least approximately*, the relation

$$2\alpha_4 - 3\alpha_3^2 - 6 = 0$$

where the symbol α_4 represents the peakedness, and the symbol α_3 represents the skewness. All of the experimental distributions were tested with the above equation and then fitted by use of tables (43) to a Pearson Type III Curve having the formula

$$Y = Y_o \left[\left(1 + \frac{\alpha_3}{2} t\right)^{\frac{4}{\alpha_3^2} - 1} \right] \left[e^{-\frac{2}{\alpha_3} t} \right]$$

and tested for goodness of fit by use of the Chi-Squared test. Sample calculations are shown in Figure 18 in the Appendix. The probability level of the fit was then determined by use of a nomograph. (40)

Plots of the Normal Curve, Log Normal Curve, and Pearson Type III Curve were superimposed individually on the histogram of each distribution.

CHAPTER V

RESULTS

Stability.—In Study A, only operator N was considered stable from the interpretation of the control charts (See Table 2). The stability of operators B, E, Q, and R were questionable and could not be conclusively classified as either stable or unstable; however, the distributions from these operators were used in further analyses. Operator B had all of the X-Bars within the three-sigma limits, but had two values outside of the limits in the range chart. Operator E had all of the X-Bars within the three-sigma limits, but had one value outside of the limits in the range chart. Operator Q had one X-Bar out of the three-sigma limits as well as one value outside of the limits in the range chart. Operator R had one X-Bar out of the three-sigma limits, but all of the range values were within the limits. Operators A, C, F, G, H, J, K, L, M, P, S, T, and U seemed to display unstable performance because of points out of control, significant runs, or significant trends.

In Study B, wherein the analysis of variance was employed (See Table 3), the 0.05 probability level was used as the basis for accepting or rejecting the null hypothesis of stability. On this basis the distributions from operators D, E, J, M, Q, R, T, and V were concluded to be stable since their between and within film shot variances were not significantly different even at the 0.20 probability level. The distribution from operator P was concluded to be stable since its between and within film

Table 2. Stability of Distributions from Operators in Study A As Determined Through Control Chart Interpretations

Operator	Stable	Remarks
A1-1A	No	Out of control
A1-2B	Questionable	All $\bar{X}$'s within limits, 2-range values out of limits
A1-3C	No	Out of control, significant runs
A1-4D		Operator changed methods during study
A1-5E	Questionable	All $\bar{X}$'s within limits, 1-range value out of limits
A1-6F	No	Out of control
A1-7G	No	Out of control, significant runs
A2-8H	No	Out of control, significant runs
A2-9J	No	Out of control, significant runs
A2-10K	No	Out of control, significant runs
A2-11L	No	Out of control
A2-12M	No	Out of control, significant runs
A3-13N	Yes	All $\bar{X}$'s and ranges within limits
A3-14P	No	Significant runs
A3-15Q	Questionable	1-$\bar{X}$ out of limits, 1-range value out of limits
A3-16R	Questionable	1-$\bar{X}$ out of limits, all range values within limits
A3-17S	No	Significant runs
A3-18T	No	Significant trend
A3-19U	No	Significant runs

Table 3. Stability of Distributions from Operators in Study B as Determined Through the Analysis of Variance

Operator	Sums of Squares B*	Sums of Squares W*	Degrees of Freedom B	Degrees of Freedom W	Mean Squares B	Mean Squares W	F Ratio	Probability Level
B1-1D	3,641	18,332	3	19	1,214	965	1.3	0.20
B1-2E	2,361	49,986	3	37	787	1,351	0.6	0.20
B1-3V	645	29,836	2,	30	323	995	0.3	0.20
B1-4B			SAMPLE SIZE TOO SMALL					
B1-5F			SAMPLE SIZE TOO SMALL					
B1-6W	28,176	22,108	4	16	7,044	1,382	5.1	<0.01
B1-7G	21,140	43,430	3	37	7,047	1,174	6.0	<0.01
B2-8H	21,676	71,901	7	58	3,097	1,240	2.5	0.04
B2-9K	27,104	49,451	7	71	3,872	696	5.5	<0.01
B2-10M	25,562	175,775	6	63	4,260	2,790	1.5	0.20
B2-11X	53,356	129,852	7	68	7,622	1,910	4.0	<0.01
B2-12Z	25,593	96,062	7	57	3,656	1,685	2.2	0.05
B2-13J	5,677	75,253	7	73	811	1,031	0.8	0.20
B3-14S	20,396	39,636	4	37	5,099	1,071	4.8	<0.01
B3-15T	9,319	76,017	7	73	1,331	1,041	1.3	0.20
B3-16P	14,241	68,076	7	65	2,034	1,047	1.9	0.12
B3-17R	3,022	50,016	7	80	432	625	0.7	0.20
B3-18Q	6,048	50,091	7	84	864	596	1.5	0.20
B3-19N	26,925	92,547	7	61	3,846	1,517	2.5	0.04

*B - Between film shots
W - Within individual film shots

shot variances were not significantly different at the 0.12 probability level. The distributions from operators G, H, K, N, S, W, X, and Z were concluded to be unstable since their between and within film shot variances were significantly different at the 0.05 probability level. The sample size was too small to perform the analysis of variance on operators B and F.

Only operators Q and R were found to be stable in both studies.

<u>Moments</u>.—Generally, the mean times and the variances in Study B were lower than those in Study A. This could possibly be explained by the fact that the complete cycle was used in Study A while the modified cycle (last element eliminated) was used in Study B. The values for skewness and peakedness did not vary significantly between the two studies. Table 4 summarizes the four moments of the distributions. The distributions for every operator in both studies were positively skewed. The combined distributions of all stable operators in Study A and in Study B were sufficiently skewed to be significantly different from normal. Three of the five stable distributions of individual operators in Study A were sufficiently peaked to be significantly different from normal, while two of the six stable distributions in Study B were sufficiently peaked to be significantly different from normal. The combined distribution of all stable operators in Study A was sufficiently peaked to be significantly different from normal, but the combined distribution of all stable operators in Study B was not.

<u>Curve-Fitting</u>.—The distribution from Operator N in Study A fitted a Normal Curve at the probability level of 0.62, while all other distri-

Table 4. Statistical Moments of Work-Time Distributions

Operator	Mean Min. x 100	Variance Min.^{2} x 100^2	Skewness	Peakedness	Sample Size	Range
			STUDY A			
A1-2B	16.2	4.19	0.15	0.760(3)*	125	10.5-23.5
A1-5E	18.2	7.28	0.93(< 1)*	0.752(< 1)*	125	12.5-29.5
A3-13N	18.7	5.97	0.27	0.785	150	11.5-26.5
A3-15Q	16.8	4.22	1.09(< 1)*	0.786	150	12.5-26.5
A3-16R	17.1	7.17	0.75(< 1)*	0.767(4)*	150	10.5-26.5
ALL	17.4	6.98	0.72(< 1)*	0.750(< 1)*	700	10.5-29.5
			STUDY B			
B2-10M	15.4	4.13	1.02(< 1)*	0.784	69	240-440
B2-13J	13.3	2.49	2.14(< 1)*	0.680(< 1)*	81	220-440
B3-15T	15.6	2.82	0.81(< 1)*	0.755(4)*	81	240-420
B3-16P	14.1	2.90	0.59(1)*	0.804	73	200-380
B3-17R	12.8	1.69	0.78(< 1)*	0.779	88	200-240
B3-18Q	13.3	1.63	1.13(< 1)*	0.778	92	230-370
ALL	14.0	3.63	1.03(< 1)*	0.787	484	200-440

NOTE: **Parameters of Study A based on distributions of entire cycle.**
Parameters of Study B based on distributions of cycle with last element removed.

*This value is significantly different from normal in that it could arise in a sample from a normally distributed universe due to chance, the per cent of the time indicated in the parentheses.

butions, including the combined distribution, in Study A fitted the Normal Curve at a probability level of less than 0.10. In Study B, the distribution from operator P fitted the Normal Curve at a probability level of 0.12. All other distributions, including the combined distribution, in Study B fitted the Normal Curve at a probability level of less than 0.10. Neither of the two operators found to be stable in both studies fitted the Normal Curve either in Study A or in Study B at a probability level of 0.10. Figure 1-13 show the Normal Curve superimposed on the respective histograms of the stable operators.

The distributions from Operators B and N in Study A fitted a Log Normal Curve at a probability level of 0.32 and 0.74 respectively, while all other distributions, including the combined distribution of stable operators, in Study A fitted the Log Normal Curve at a probability level of less than 0.10. In Study B, operator P fitted the Log Normal Curve at a probability level of 0.45, operator R at a probability level of 0.28, operator T at a probability level of 0.19, and operator M at a probability level of 0.16. The distributions from operators J and Q as well as the combined distribution fitted the Log Normal Curve at a probability level of less than 0.10. Of the two operators found to be stable in both studies, neither fitted the Log Normal Curve in Study A, but operator R fitted the Log Normal Curve in Study B. Figure 1-13 show the Log Normal Curve superimposed on the respective histograms of the stable operators.

The Poisson Curve did not typify any of the distributions since none of the distributions had means and variances approximately equivalent.

In the test equation for a Pearson Type III Curve set forth by Kenney (42), all but one of the distributions indicated a Pearson Type III possibility (See Table 5). Therefore each was fitted to this curve. The distribution of operator Q in Study A fitted a Pearson Type III Curve at a probability level of 0.84, operator M at a probability level of 0.70, operator R at a level of 0.34, the combined distribution at a level of 0.20. The distributions of operators B and E fitted the Pearson Type III Curve at a probability level of less than 0.10. In Study B, the distribution of operator P fitted this curve at a probability level of 0.60, operator M at a probability level of 0.50, operator R at a level of 0.38, operator Q at a level of 0.15, and the combined distribution at a level of 0.36. In Study B, only the distribution from operator J did not fit the Pearson Type III Curve. Its skew was too great to attempt a fit. Both of the operators who were stable in the two studies fitted the Pearson Type III Curve in Study A as well as in Study B. Figures 1-13 show the Pearson Type III Curve superimposed on the respective histograms of the stable operators.

Table 6 summarizes the goodness of fit of the Normal Curve, Log Normal Curve, and the Pearson Type III Curve for all the distributions.

Summary of Results

<u>Stability</u>.—In Study A, one distribution was considered to be stable, while the stability of four distributions was questionable. The other fourteen distributions were considered to be unstable.

In Study B, nine distributions were considered to be stable, eight distributions were considered to be unstable.

<u>Moments</u>.—All work-time distributions were positively skewed. Four of the six distributions in Study A and all of the seven distributions in Study B

Table 5. Preliminary Test for Pearson Type III Curve

Operator	α_4	α_3	$2\alpha_4 - 3\alpha_3^2 - 6$*
		STUDY A	
A1-2B	4.13	0.15	2.20
A1-5E	4.64	0.93	0.68
A3-13N	3.23	0.27	0.24
A3-15Q	4.79	1.09	0.01
A3-16R	3.97	0.75	0.26
ALL	3.91	0.72	0.26
		STUDY B	
B2-10M	3.76	1.02	-1.60
B2-13J	204.2	2.14	388.70
B3-15T	3.36	0.81	-1.25
B3-16P	3.00	0.59	-1.05
B3-17R	3.75	0.78	-0.33
B3-18Q	4.94	1.13	0.05
ALL	4.32	1.03	0.54

*Kenney (42) states that if a Pearson Type III is to represent an experimental distribution, the moments of that distribution should satisfy, at least approximately, the relation

$$2\alpha_4 - 3\alpha_3^2 - 6 = 0$$

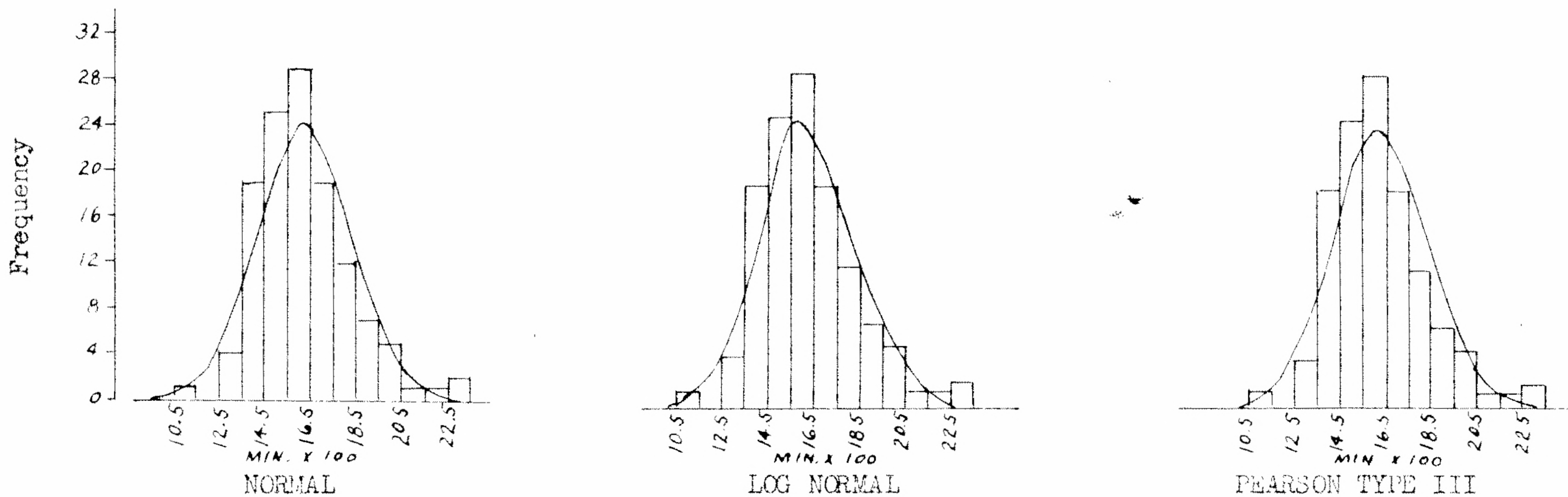
Table 6. Probability Level of Goodness of Fit for the Obtained Work-Time Distributions to the Normal, Log Normal, and Pearson Type III Curves

Operator	Sample Size	Normal X^2	Normal D.F.	Normal Prob.	Log Normal X^2	Log Normal D.F.	Log Normal Prob.	Pearson Type III X^2	Pearson Type III D.F.	Pearson Type III Prob.
				STUDY A						
A1-2B	125	11.3	5	0.04	5.9	5	0.32	9.6	4	0.04
A1-5E	125	17.2	6	<0.01	12.7	6	0.04	15.8	5	<0.01
A3-13N	150	4.3	6	0.62	3.4	6	0.74	3.0	5	0.70
A3-15Q	150	19.6	5	<0.01	10.3	5	0.06	1.4	4	0.84
A3-16R	150	15.9	6	0.01	17.7	7	0.01	5.5	5	0.34
ALL	700	62.6	9	<0.01	24.3	11	0.01	12.2	9	0.20
				STUDY B						
B2-10M	69	11.2	4	0.02	6.7	4	0.16	2.4	3	0.50
B2-13J	81	12.6	2	<0.01	7.6	2	0.02	- -	- -	- -
B3-15T	81	7.3	3	0.06	4.9	3	0.19	2.9	2	0.24
B3-16P	73	6.0	3	0.12	3.7	4	0.45	1.9	3	0.60
B3-17R	88	6.8	3	0.08	3.9	3	0.28	2.0	2	0.38
B3-18Q	92	15.4	5	<0.01	11.4	5	0.04	6.6	4	0.15
ALL	484	73.3	6	<0.01	36.7	7	<0.01	6.2	6	0.36

NOTE: X^2 - abbreviation for Chi-Squared
D.F. - abbreviation for degrees of freedom
Prob. - abbreviation for probability level

Operator A1-2B

n = 125



Probability Level—0.04   Probability Level—0.32   Probability Level—0.04

Figure 1. Theoretical Curves Superimposed on Histograms of Operator A1-2B

Operator A1-5E

n = 125

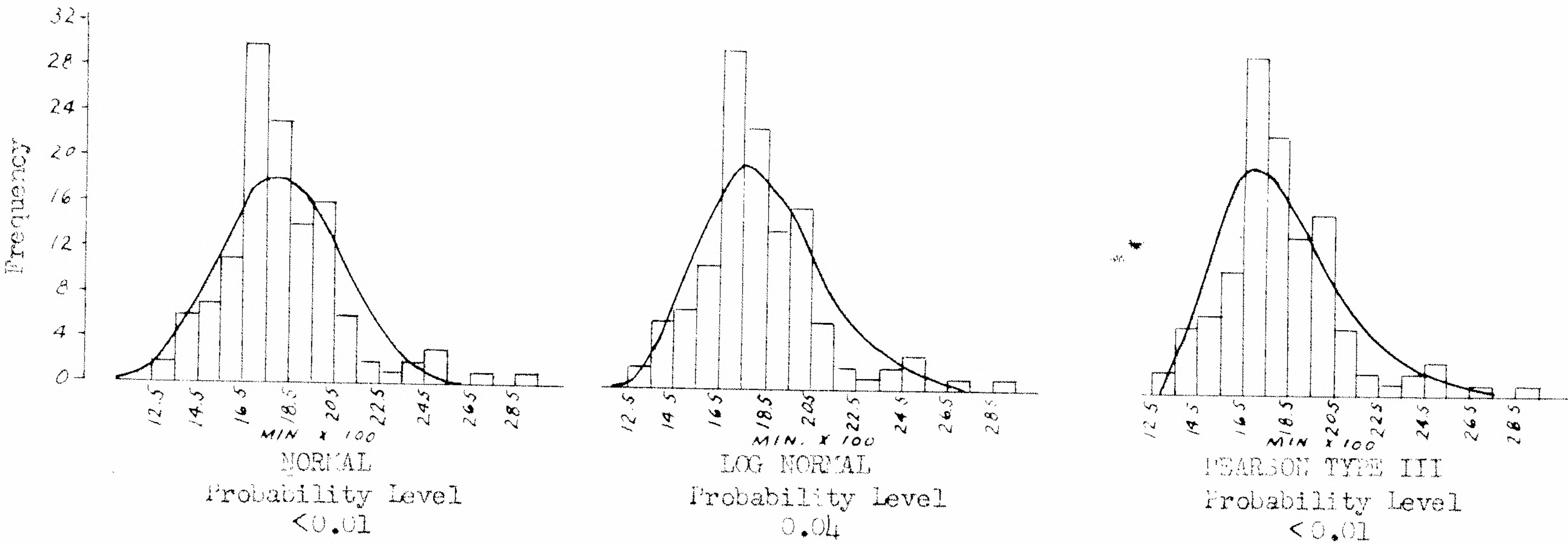
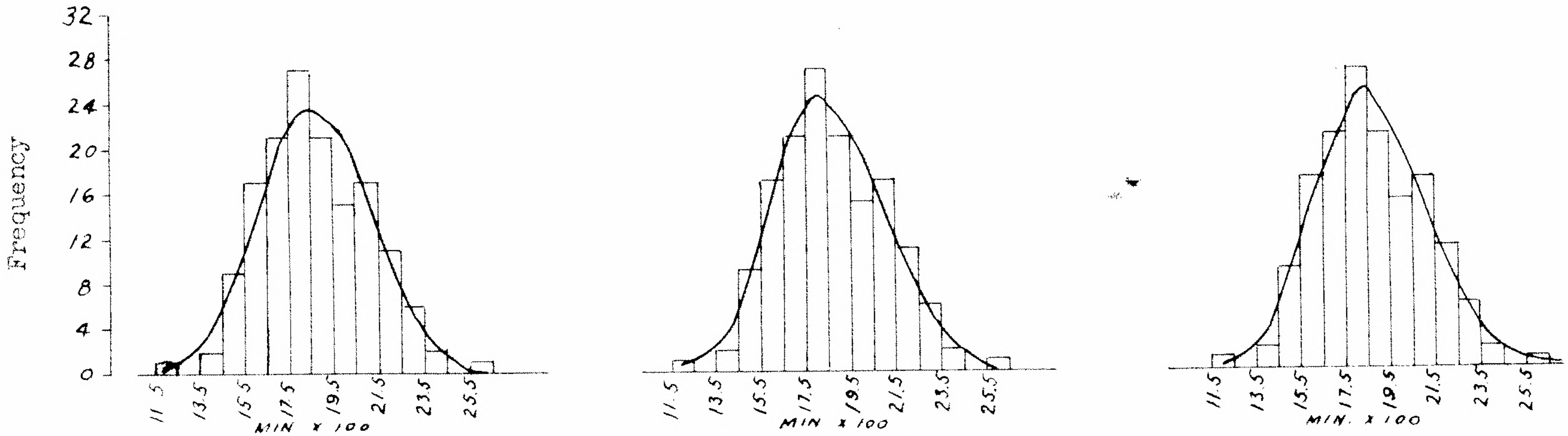


Figure 2. Theoretical Curves Superimposed on Histograms of Operator A1-5E

Operator A3-13N

n = 150



NORMAL	LOG NORMAL	PEARSON TYPE III
Probability Level 0.62	Probability Level 0.74	Probability Level 0.70

Figure 3. Theoretical Curves Superimposed on Histograms of Operator A3-13N

Operator 43-150

n = 150

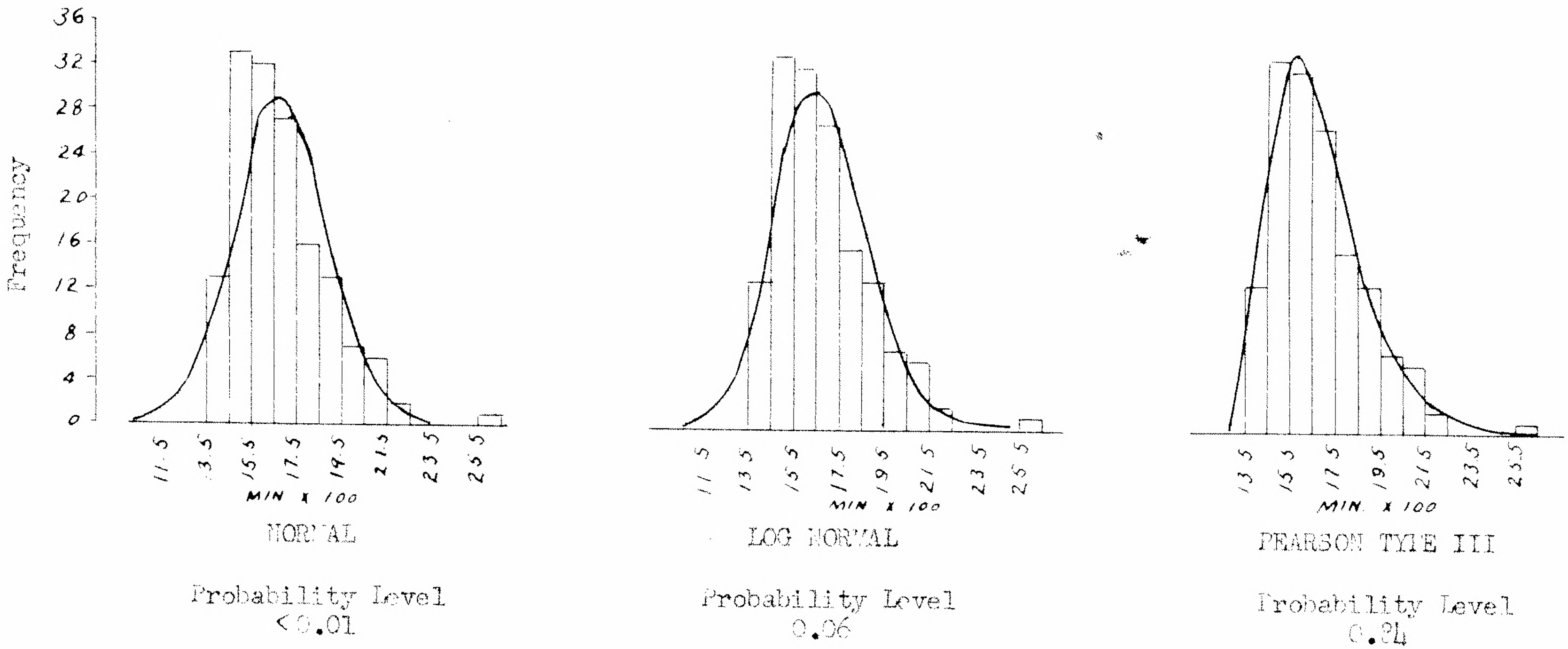
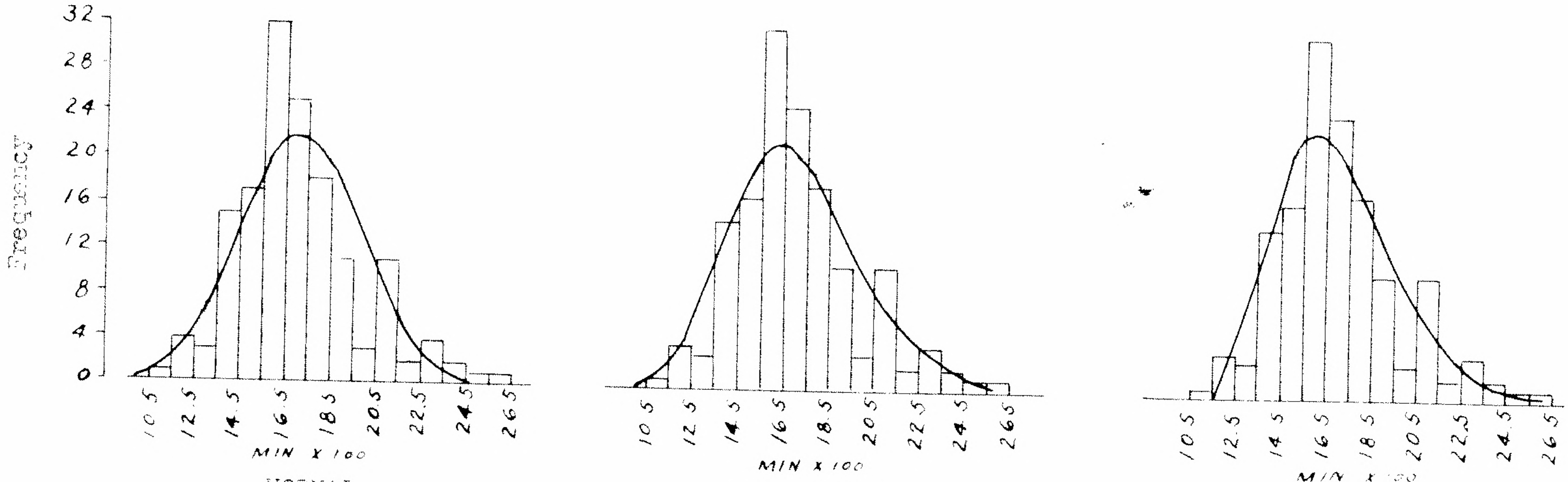


Figure 4. Theoretical Curves Superimposed on Histograms of Operator 43-150

Operator A3-16R

n = 150



NORMAL

Probability Level
0.01

LOG NORMAL

Probability Level
0.21

PEARSON TYPE III

Probability Level
0.34

Figure 5. Theoretical Curves Superimposed on Histograms of Operator A3-16R

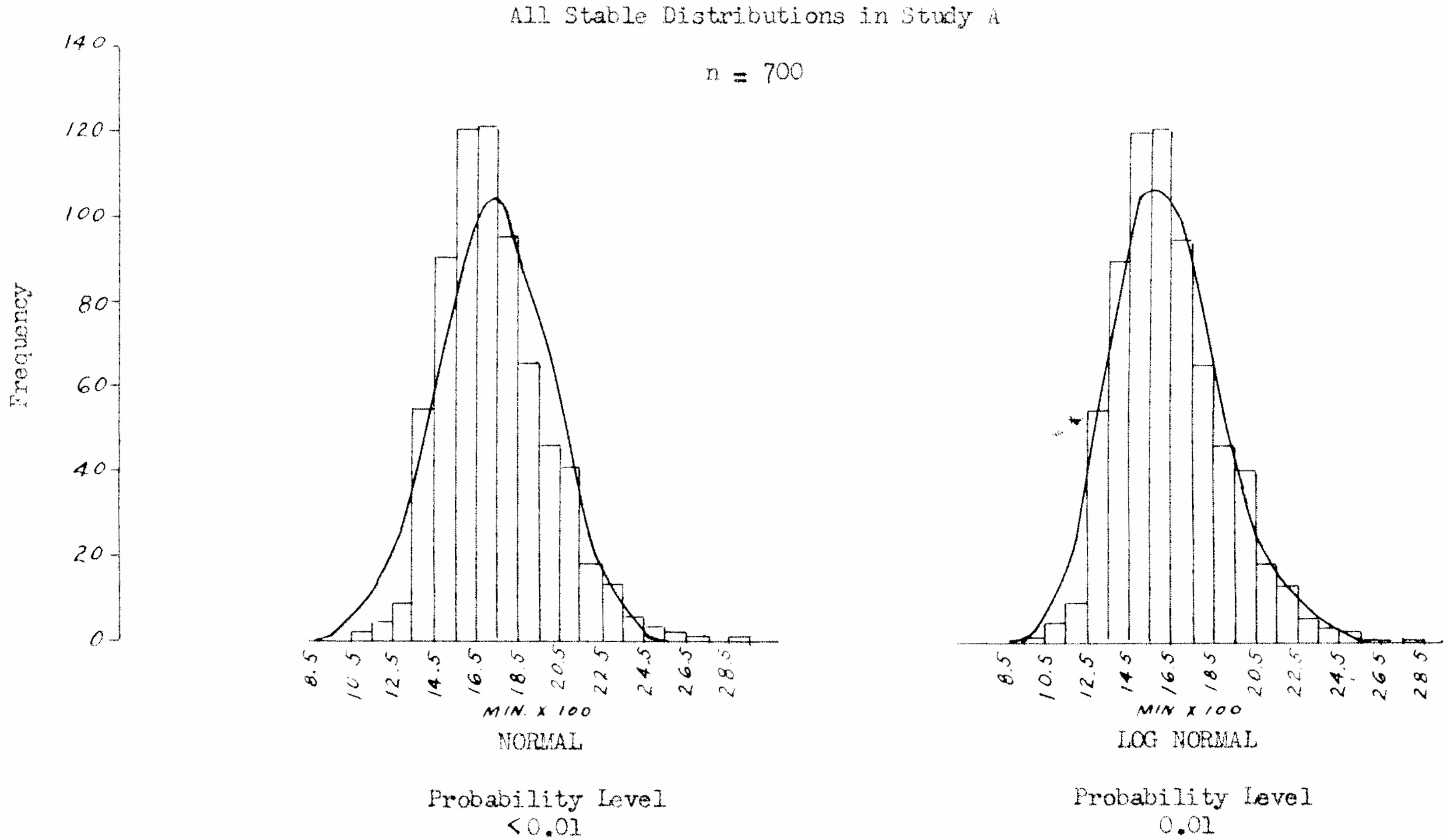
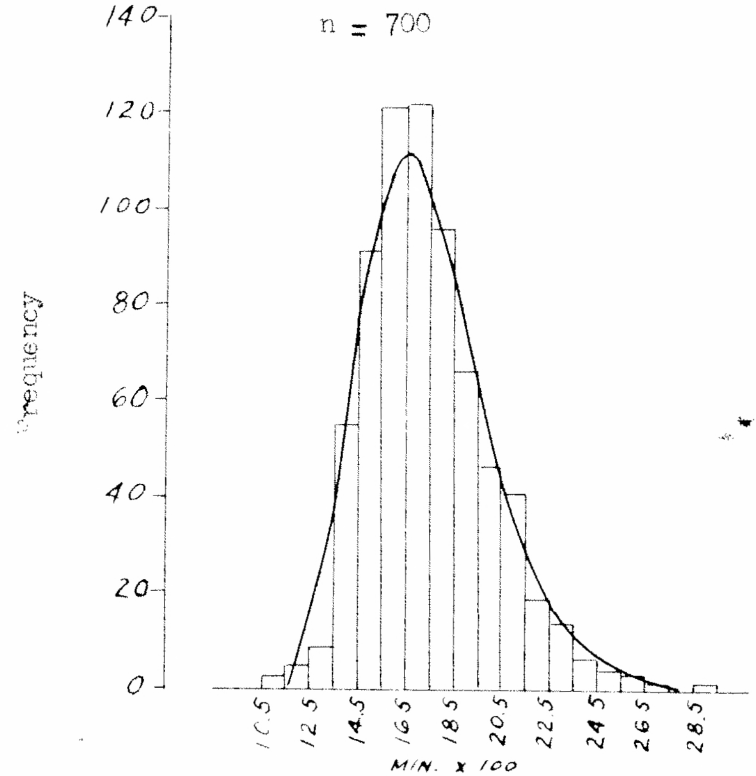


Figure 6. Theoretical Curves Superimposed on Histograms of All Stable Operators in Study A

All Stable Distributions in Study A (continued)



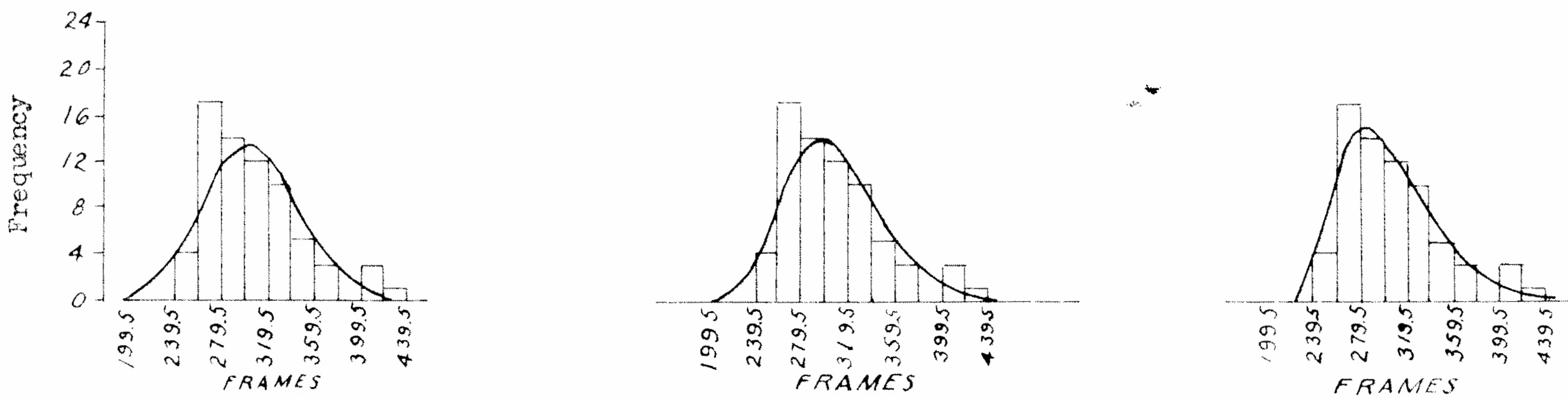
PEARSON TYPE III

Probability Level
0.20

Figure 6. Theoretical Curves Superimposed on Histograms of All Stable Operators in Study A

Operator B2-10M

n = 69

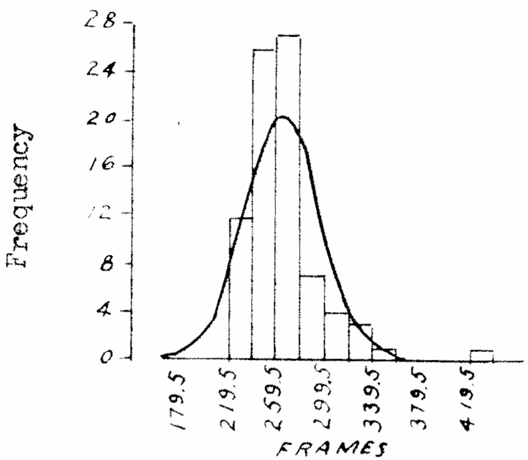


NORMAL	LOG NORMAL	PEARSON TYPE III
Probability Level 0.02	Probability Level 0.16	Probability Level 0.50

Figure 7. Theoretical Curves Superimposed on Histograms of Operator B2-10M

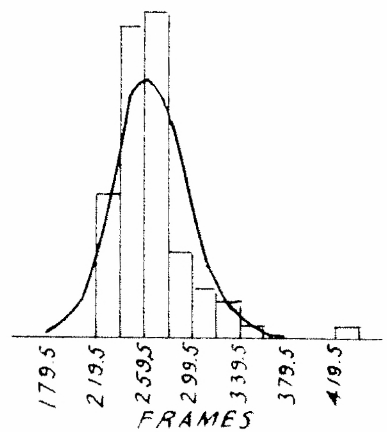
Operator B2-13J

n = 81



NORMAL

Probability Level
< 0.01



LOG NORMAL

Probability Level
0.02

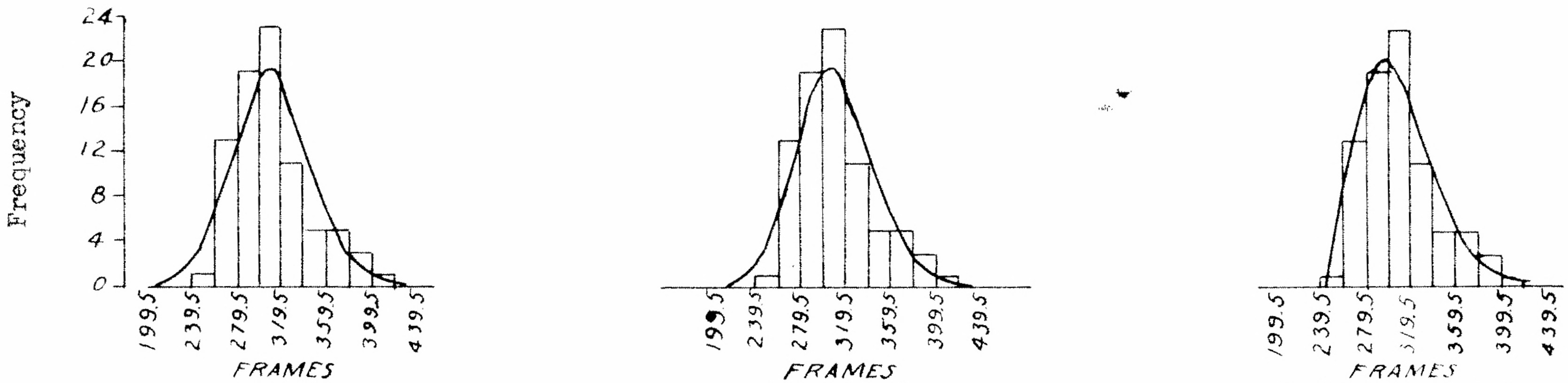
Amount of Skew
too Great to Fit
a Pearson Type III
Curve

PEARSON TYPE III

Figure 8. Theoretical Curves Superimposed on Histograms of Operator B2-13J

Operator B3-15T

n = 81



NORMAL

Probability Level
0.06

LOG NORMAL

Probability Level
0.19

PEARSON TYPE III

Probability Level
0.24

Figure 9. Theoretical Curves Superimposed on Histograms of Operator B3-15T

Operator B3-16P

n = 73

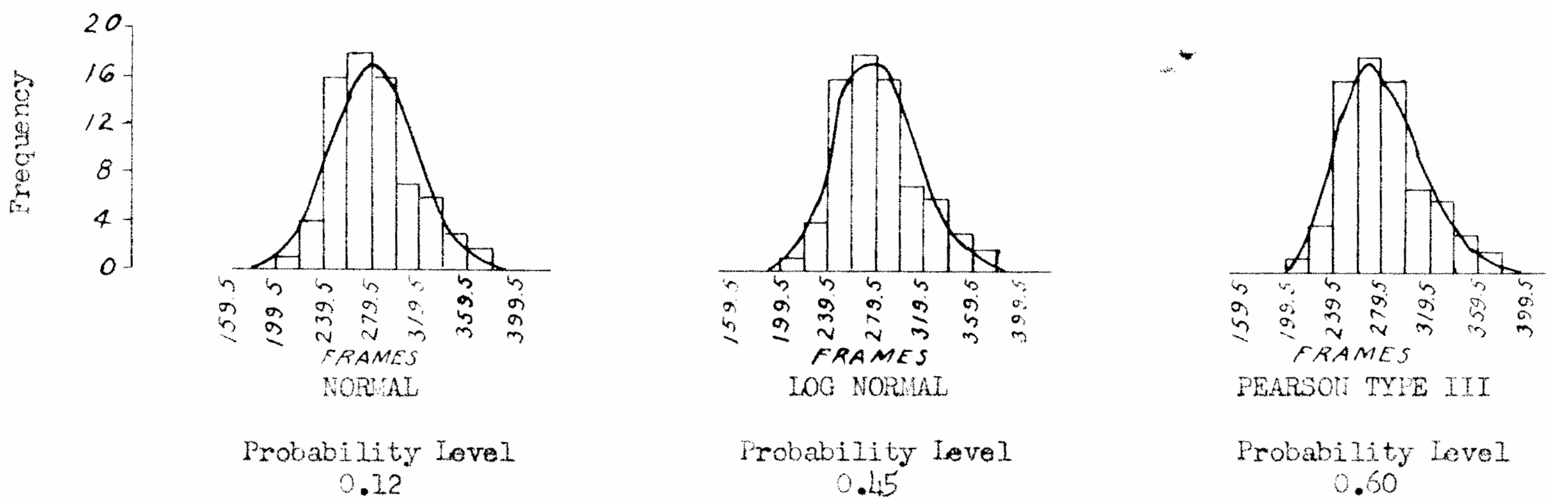
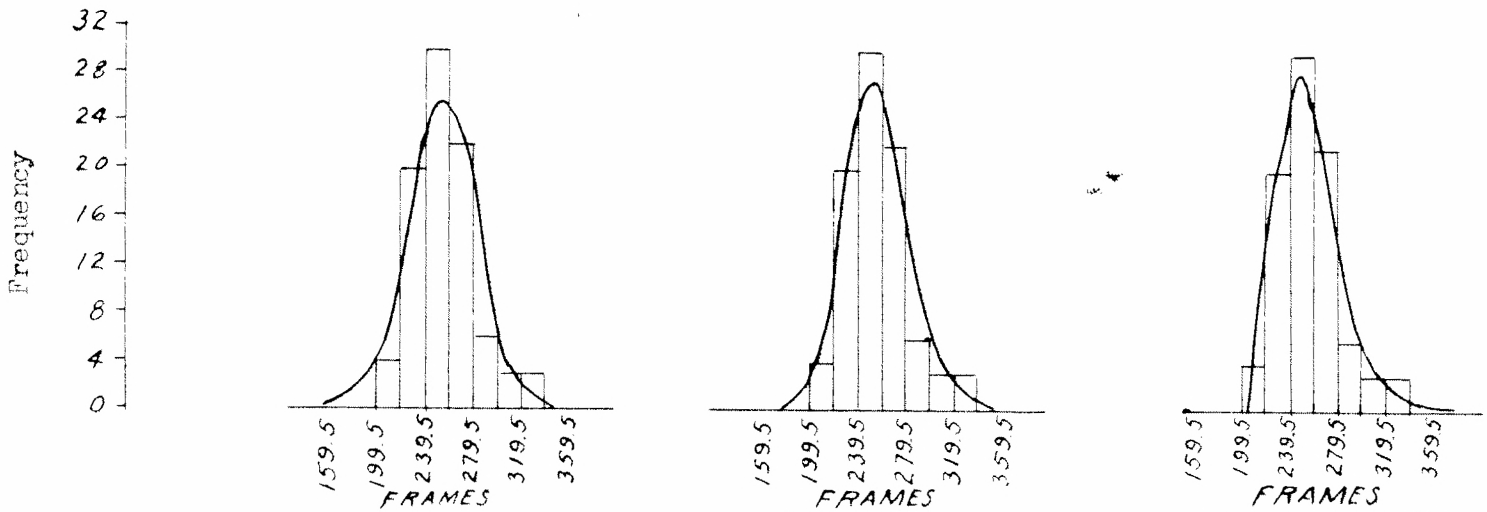


Figure 10. Theoretical Curves Superimposed on Histograms of Operator B3-16P

Operator B3-17R

n = 88

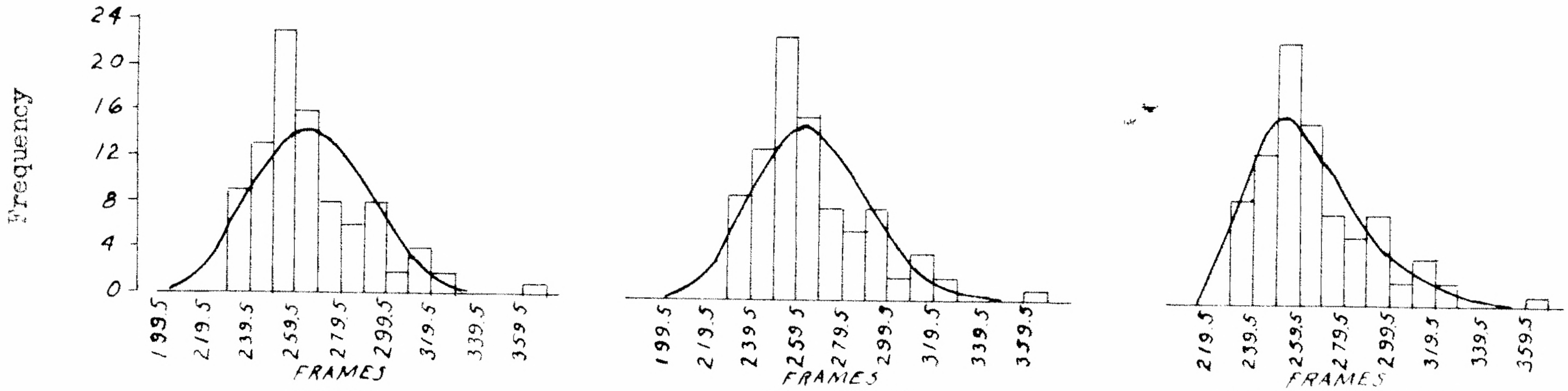


NORMAL	LOG NORMAL	PEARSON TYPE III
Probability Level 0.08	Probability Level 0.28	Probability Level 0.38

Figure 11. Theoretical Curves Superimposed on Histograms of Operator B3-17R

Operator B3-18Q

n = 92

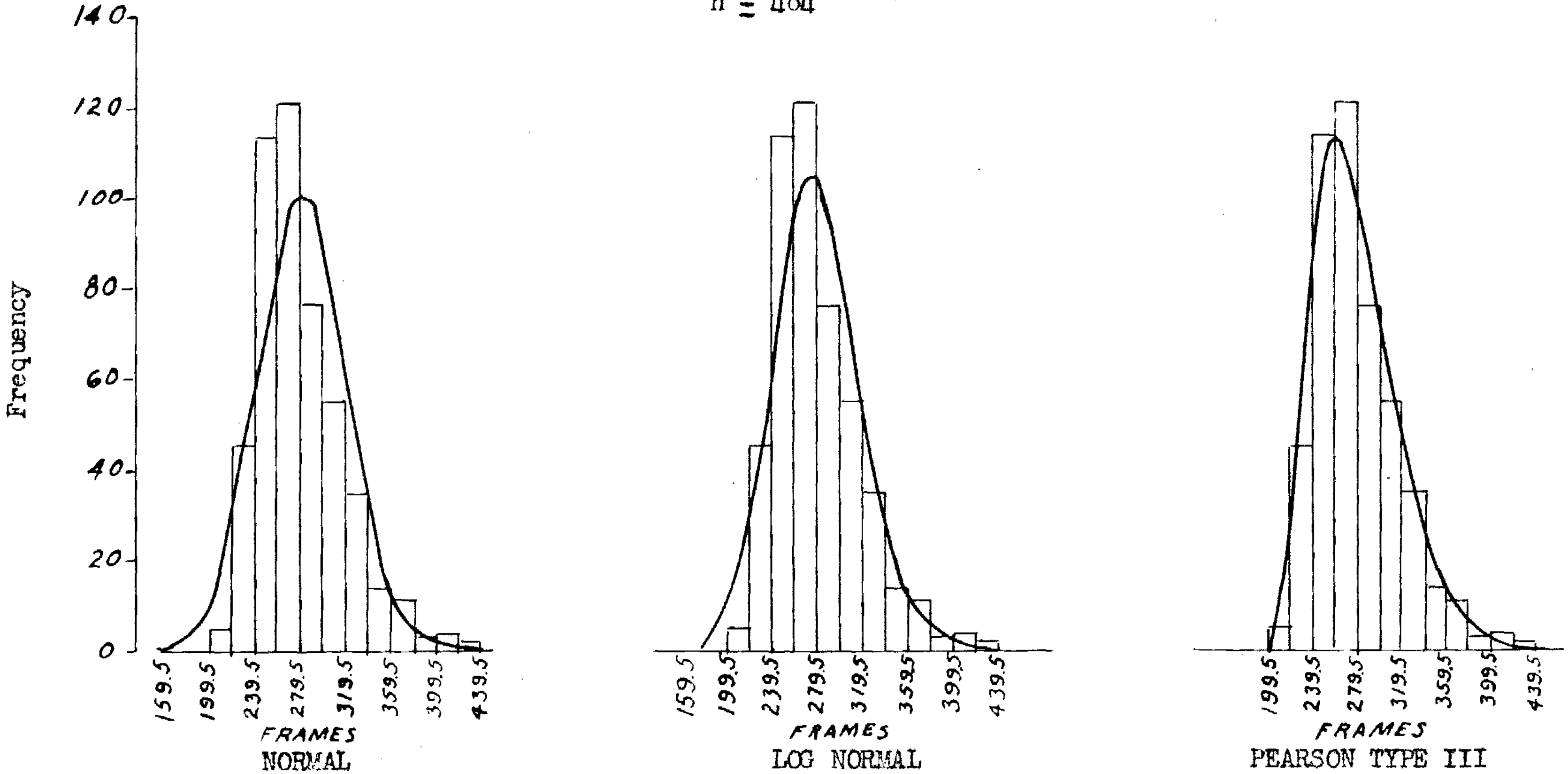


NORMAL	LOG NORMAL	PEARSON TYPE III
Probability Level <0.01	Probability Level 0.04	Probability Level 0.15

Figure 12. Theoretical Curves Superimposed on Histograms of Operator B3-18Q

All Stable Distributions in Study B

n = 484



Probability Level
<0.01

Probability Level
<0.01

Probability Level
0.36

Figure 13. Theoretical Curves Superimposed on Histograms of All Stable Operators in Study B

were sufficiently skewed to be considered significantly different from normal.

Four of the six distributions in Study A and two of the seven distributions in Study B were sufficiently peaked to be considered significantly different from normal.

Curve-Fitting.—One distribution from each study fitted the Normal Curve at a probability level greater than 0.10. Two distributions from Study A and four distributions from Study B fitted the Log Normal Curve at a probability level greater than 0.10. None of the distributions from either study fitted the Poisson Curve. Four distributions from Study A and six distributions from Study B fitted the Pearson Type III Curve at a probability level of greater than 0.10. The Pearson Type III Curve was the only one, of those tried, which fitted the combined distribution of all the stable operators in each study. The Pearson Type III Curve was the only one of those tried which represented, in Study A and in Study B, the distributions of the two operators who were found to be stable in both studies.

See Table 7 for a summary of all results.

Table 7. Summary of Results

Operator	Stability Level Study A*	Stability Level Study B	Skewness Probability Difference from Normal St. A	Skewness Probability Difference from Normal St. B	Peakedness Probability Difference from Normal St. A	Peakedness Probability Difference from Normal St. B	Curve Fitting Probability of Goodness of Fit Normal St. A	Normal St. B	Log-Normal St. A	Log-Normal St. B	Pearson III St. A	Pearson III St. B
B	Possibly	**	>0.05	**	0.03	**	0.04	**	0.32	**	0.04	**
E	Possibly	0.20	<0.01	***	<0.01	***	<0.01	***	0.04	***	<0.01	***
J	No	0.20	****	<0.01	****	<0.01	****	0.01	****	0.02	****	--
M	No	0.20	****	<0.01	****	>0.10	****	0.02	****	0.16	****	0.50
N	Yes	0.04	>0.05	****	>0.10	****	0.62	****	0.74	****	0.70	****
P	No	0.12	****	0.01	****	>0.10	****	0.12	****	0.45	****	0.60
Q	Possibly	0.20	<0.01	<0.01	>0.10	>0.10	<0.01	<0.01	0.06	0.04	0.84	0.15
R	Possibly	0.20	<0.01	<0.01	0.04	>0.10	0.01	0.08	0.01	0.28	0.34	0.38
T	No	0.20	****	<0.01	****	0.04	****	0.06	****	0.19	****	0.24
ALL			<0.01	<0.01	<0.01	>0.10	<0.01	<0.01	0.01	<0.01	0.20	0.36

*Stability for distributions from Study A determined by control charts. Reasons for instability are significant runs, trends, or more than one point out of control.

**Operator not in this study.

***Sample size not adequate.

****Values not included because of instability of distributions.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The present investigation is mainly concerned with the determination of the statistical characteristics of work-time distributions obtained from periodic observation of a number of operators performing a short cycled manually controlled operation. The data in this study are taken from a larger group of data obtained from a long-range research project begun in 1951 by Doctors Lehrer and Moder at the Georgia Institute of Technology. The overall project is concerned with the investigation of the statistical characteristics of work-time phenomena. The selection of data from the project was made on the basis of stability of operator performance through the entire work period.

In view of the experimental situation, it seems reasonable to conclude that there is a theoretical work-time distribution which typifies work-time phenomena for this type of activity. This theoretical work-time distribution appears to have the following characteristics:

1. It differs significantly from the Normal Curve.
2. It is positively skewed.
3. Its peakedness is greater than that of the Normal Curve.
4. It can be reasonably approximated by a Pearson Type III Curve.

The general hypothesis that there is a theoretical work-time distribution has seldom been challenged. It has been generally assumed that this theoretical distribution closely approximates the Normal Curve. Data previously obtained in the research project from which this thesis data are drawn indicates that the theoretical distribution differs from the Normal Curve. The present investigation clearly indicates that this difference is significantly greater than can be explained by chance variations.

The various curve functions that were fitted to the thesis data indicate that the Normal Curve and the Poisson Curve do not adequately fit the data, that the Log Normal Curve does fit part of the data, and that the Pearson Type III Curve fits a substantial portion of the data.

Specific conclusions are:

1. For the operation studied, the work-time distributions can be typified by a Pearson Type III Curve.

2. For the operation studied, the work-time distributions can not adequately be represented by either a Normal Curve or a Poisson Curve.

3. For the operation studied, the work-time distributions may possibly be represented by a Log Normal Curve.

The above conclusions should be viewed in light of the limitations of the present investigation. The sample sizes range for individual work-time distributions from 125 to 150 in Study A and from 70 to 92 in Study B. Because of the criteria of statistical stability, the number of distributions analyzed is limited to six from Study A and seven from Study B. A further limitation on the conclusions is that the operation is manually controlled and the cycle times are relatively short. Also, the operators seem to be highly motivated. In Study A, the shortness of the overall cycle time in conjunction with the stop watch limitations prevent the elimination of all foreign elements to the standard method. The foreign elements are more readily eliminated with the micromotion study since the film can be viewed repeatedly and at a slower rate than the original operation.

It is therefore recommended that another film be taken of the same operation in which a large sample size is recorded for each operator in as short a time interval as possible in order to minimize variation of operator pace during the observation period. The film should be taken in the middle of the week and early in the shift in order to eliminate the work decrement factor from the data.

It is further recommended that the distributions from the proposed film be tested for stability. If found to be stable, the distributions should be fitted to the Normal, Log Normal, Pearson Type III, and other suitable curves to test the hypothesis formulated at the conclusion of this thesis that the work-time distributions of statistically stable operators on a short cycled manually controlled operation can be typified by the Pearson Type III Curve.

APPENDIX

Table 8. Work-Time Frequency Distributions from Study A

FIRST SHIFT

Cycle Time Min. x 100	Operator A1-1A	A1-2B	A1-3C	A1-4D	A1-5E	A1-6F	A1-7G
11		1					
12							1
13		4			2	1	5
14		19	5		6	5	13
15	1	25	15	1	7	9	24
16	10	29	28		11	12	26
17	22	19	15	1	30	12	25
18	16	12	14	1	23	8	13
19	28	7	16		14	18	8
20	12	5	5	5	16	7	3
21	11	1	11	4	6	8	1
22	10	1	4	5	2	9	1
23	4	2	1	12	1	7	
24	3		1	12	2	4	1
25	5			16	3	2	
26				8		3	
27	3		3	14	1	2	1
28			1	13		4	2
29			3	8	1	2	
30				7		1	
31				5		1	
32			1	1		3	
33				3		2	
34			1	1			
35				1			1
36				3			
37				1		1	
38			1	1		1	
39						1	
40				2		1	
41						1	
TOTALS	125	125	125	125	125	125	125

Table 8. Work-Time Frequency Distributions from Study A

(Continued)

SECOND SHIFT

Cycle Time Min. x 100	A2-8H	A2-9J	Operator A2-10K	A2-11L	A2-12M
11					
12		4	4	13	
13	2	12	25	27	1
14	4	42	40	35	8
15	10	55	50	45	22
16	16	64	36	54	26
17	25	39	31	35	43
18	21	19	24	23	44
19	36	13	13	10	33
20	25	4	18	10	25
21	20	7	9	7	25
22	17	5	8	4	14
23	23	5	4	5	4
24	22			1	7
25	10		4	3	2
26	9	2	1	1	7
27	10		2	1	4
28	4	1	3		3
29	6			1	2
30	2		1		1
31	1	1			3
32	2		1		
33	1				1
34	1		1		
35	1				
36					
37					
38		1			
39		1			
40					
41					
TOTALS	190	275	275	275	275

Table 8. Work-Time Frequency Distributions from Study A

(Continued)

THIRD SHIFT

Cycle Time Min. x 100	A3-13N	A3-14P	A3-15Q	Operator A3-16R	A3-17S	A3-18T	A3-19U
11				1			
12	1			4			
13		1		3			
14	2	2	13	15	2	1	
15	9	6	33	17	2	1	1
16	17	8	32	32	3	12	2
17	21	13	27	25	12	13	5
18	27	24	16	18	14	34	11
19	21	22	13	11	23	26	12
20	15	24	7	3	23	18	15
21	17	13	6	11	17	1	16
22	11	16	2	2	11	8	15
23	6	10		4	9	9	24
24	2	7		2	9	1	13
25		2		1	4	1	3
26	1		1	1		2	10
27		1			3	1	5
28		1			6		8
29					1	1	
30					1		2
31						1	3
32							
33							1
34							1
35							1
36							
37							
38							
39							
40							
41							
TOTALS	150	150	150	150	140	140	150

Table 9. Work-Time Distributions From Operators in Study No. B

FIRST SHIFT

Cycle Times in Frames	Operator B1-1D	B1-2E	B1-3U	B1-4B	B1-5F	B1-6W	B1-7G
229.5-239.5			1				
239.5-249.5		3	1				3
249.5-259.5		9	2				4
259.5-269.5	1	5	5				8
269.5-279.5	4	4	6	3	2		5
279.5-289.5	1	6	4	1			3
289.5-299.5		2	2		1		3
299.5-309.5	2	2	5	1			3
309.5-319.5	7	4	1	1			2
319.5-329.5	4	2	1				4
329.5-339.5	1		2	1			1
339.5-349.5		1	1		1		2
349.5-359.5	1		2		2		2
359.5-369.5	1			1			
369.5-379.5		2			1		
379.5-389.5							
389.5-399.5		1					
399.5-409.5							
409.5-419.5	1					1	
419.5-429.5					1	1	
429.5-439.5							
439.5-449.5						1	
449.5-459.5						2	
459.5-469.5						1	
469.5-479.5						4	
479.5-489.5						1	
489.5-499.5						4	
499.5-509.5						1	
529.5-539.5						2	
549.5-559.5						1	
559.5-569.5						1	
629.5-639.5						1	
TOTALS	23	41	33	8	8	21	41

Table 9. Work-Time Distributions From Operators in Study No. B

(Continued)

SECOND SHIFT

Cycle Times in Frames	Operator B2-1H	B2-2K	B2-3M	B2-4X	B2-5Z	B2-6J
209.5-219.5		1		1		
219.5-229.5		4		2		3
229.5-239.5		7		3		9
239.5-249.5	3	17	1	10		16
249.5-259.5	1	9	3	12		10
259.5-269.5	2	10	6	7	1	16
269.5-279.5	6	8	11	4	1	11
279.5-289.5	10	5	11	15		4
289.5-299.5	10	4	3	5	1	3
299.5-309.5	13	3	7	3	7	3
309.5-319.5	2	6	5	4	4	1
319.5-329.5	3	1	6		8	1
329.5-339.5	4	2	4	1	12	2
339.5-349.5	4		5	2	6	
349.5-359.5	2	2		3	4	1
359.5-369.5	1		2		5	
369.5-379.5	1		1		6	
379.5-389.5	1				3	
389.5-399.5					2	
399.5-409.5	1		3	1	1	
409.5-419.5	1					
419.5-429.5	1			2	1	
429.5-439.5			1			1
439.5-449.5					1	
499.5-509.5				1	1	
509.5-519.5					1	
TOTALS	66	79	69	76	65	81

Table 9. Work-Time Distributions From Operators in Study No. B

(Continued)

THIRD SHIFT

Cycle Times in Frames	Operator B3-1R	B3-2S	B3-3O	B3-4Q	B3-5P	B3-6N
209.5-219.5			1	4		
219.5-229.5			1	8		
229.5-239.5			3	12	9	
239.5-249.5			9	16	13	1
249.5-259.5		1	7	14	23	1
259.5-269.5		2	9	13	16	1
269.5-279.5		11	9	9	8	1
279.5-289.5	1	10	8	4	6	7
289.5-299.5	1	9	8	2	8	9
299.5-309.5	4	10	4	2	2	5
309.5-319.5	4	13	3	1	4	6
319.5-329.5	9	1	3	1	2	8
329.5-339.5	2	10	3	2		12
339.5-349.5	5	2				2
349.5-359.5	3	3	3			3
359.5-369.5	3	4	2		1	3
369.5-379.5	3	1				2
379.5-389.5	2	3				1
389.5-399.5	2					3
399.5-409.5						1
409.5-419.5	1	1				1
419.5-429.5						
429.5-439.5	1					
439.5-449.5						1
449.5-459.5						1
459.5-469.5						
469.5-479.5	1					
TOTALS	42	81	73	88	92	69

Table 10. Stable Work-Time Frequency Distributions From Study A

Cycle Time Min. x 100	Operator A1-2B	A1-5E	A3-13N	A3-15Q	A3-16R	ALL
11	1				1	2
12			1		4	5
13	4	2			3	9
14	19	6	2	13	15	55
15	25	7	9	33	17	91
16	29	11	17	32	32	121
17	19	30	21	27	25	122
18	12	23	27	16	18	96
19	7	14	21	13	11	66
20	5	16	15	7	3	46
21	1	6	17	6	11	41
22	1	2	11	2	2	18
23	2	1	6		4	13
24		2	2		2	6
25		3			1	4
26			1	1	1	3
27		1				1
28						
29		1				1
TOTALS	125	125	150	150	150	700

Table 11. Stable Work-Time Frequency Distributions From Study B

Cycle Time in Frames	Operator B2-10M	B2-13J	B3-15T	B3-16P	B3-17R	B3-18Q	ALL
209.5-219.5				1	4		5
219.5-229.5		3		1	8		12
229.5-239.5		9		3	12	9	33
239.5-249.5	1	16		9	16	13	55
249.5-259.5	3	10	1	7	14	23	58
259.5-269.5	6	16	2	9	13	16	62
269.5-279.5	11	11	11	9	9	8	59
279.5-289.5	11	4	10	8	4	6	43
289.5-299.5	3	3	9	8	2	8	33
299.5-309.5	7	3	10	4	2	2	28
309.5-319.5	5	1	13	3	1	4	27
319.5-329.5	6	1	1	3	1	2	14
329.5-339.5	4	2	10	3	2		21
339.5-349.5	5		2				7
349.5-359.5		1	3	3			7
359.5-369.5	2		4	2		1	9
369.5-379.5	1		1				2
379.5-389.5			3				3
389.5-399.5							
399.5-409.5	3						3
409.5-419.5			1				1
419.5-429.5							
429.5-439.5	1	1					2
TOTALS	69	81	81	73	88	92	484

Table 12. Sums and Sums of Squares of Cycle Times of Raw Data by Film Shots

Operator	Shot	Sums of Cycle Times in Frames	Sums of Squares of Cycle Times in Frames	Number Cycles Per Shot
B1-1D	4	1,604	524,532	5
	5	1,966	649,252	6
	6	1,876	588,200	6
	7	1,767	522,049	6
B1-2E	4	3,485	1,027,623	12
	5	3,263	991,849	11
	6	2,768	771,994	10
	7	2,260	643,178	8
B1-3V	4	3,434	994,546	12
	6	3,171	923,835	11
	7	2,966	887,980	10
B1-4B	7	2,430	745,824	8
B1-5F	5	1,196	361,670	4
	7	1,507	571,823	4
B1-6W	3	870	379,332	2
	4	2,304	1,064,088	5
	5	2,880	1,383,638	6
	6	1,617	871,787	3
	7	2,676	1,449,554	5
B1-7G	4	2,786	778,962	10
	5	2,995	823,935	11
	6	3,238	963,804	11
	7	2,996	1,018,850	9
B2-8H	1	2,378	813,814	7
	2	2,777	864,015	9
	3	2,163	683,881	7
	4	2,434	750,164	8
	5	2,486	690,238	9
	6	2,639	777,753	9
	7	1,807	546,731	6
	8	3,577	1,186,801	11

Table 12. Sums and Sums of Squares of Cycle Times of Raw Data by Film Shots

(Continued)

Operator	Shot	Sums of Cycle Times in Frames	Sums of Squares of Cycle Times in Frames	Number Cycles Per Shot
B2-9K	1	2,963	892,565	10
	2	2,589	678,663	10
	3	3,520	893,540	14
	4	1,873	503,377	7
	5	3,005	825,525	11
	6	2,483	621,615	10
	7	2,314	597,374	9
	8	2,415	732,633	8
B2-10M	1	2,888	949,414	9
	2	2,055	606,567	8
	3	2,419	737,449	8
	4	2,752	870,752	9
	5	3,463	1,004,933	12
	6	3,701	1,151,363	12
	8	3,823	1,241,605	12
B2-11X	1	2,855	823,301	10
	2	3,417	990,655	12
	3	3,374	973,822	12
	4	3,327	930,127	12
	5	2,725	752,951	10
	6	2,812	733,090	11
	7	875	258,429	3
	8	2,208	855,802	6
B2-12Z	1	2,010	830,934	5
	2	2,623	1,002,971	7
	3	2,813	992,425	8
	4	3,668	1,244,430	11
	5	3,060	1,049,518	9
	6	3,048	1,044,228	9
	7	2,662	886,870	8
	8	2,730	937,864	8

Table 12. Sums and Sums of Squares of Cycle Times of Raw Data by Film Shots

(Continued)

Operator	Shot	Sums of Cycle Times in Frames	Sums of Squares of Cycle Times in Frames	Number Cycles Per Shot
B2-13J	1	3,245	886,989	12
	2	2,273	578,189	9
	3	2,890	766,078	11
	4	2,341	615,927	9
	5	2,395	644,159	9
	6	2,885	759,551	11
	7	2,564	766,224	9
	8	2,942	789,198	11
B3-14S	1	3,398	1,160,058	10
	2	3,011	1,012,957	9
	3	3,676	1,245,994	11
	5	2,023	825,731	5
	6	2,389	819,173	7
B3-15T	1	2,615	861,943	8
	2	3,222	1,041,398	10
	3	3,145	995,147	10
	4	3,160	1,014,238	10
	5	2,902	951,898	9
	6	3,557	1,059,351	12
	7	3,013	918,493	10
	8	3,621	1,104,661	12
B3-16P	1	2,431	749,289	8
	2	2,769	769,197	10
	3	2,994	824,128	11
	4	1,216	370,254	4
	5	3,063	862,831	11
	6	1,608	436,084	6
	7	2,975	903,975	10
	8	3,452	927,902	13

Table 12. Sums and Sums of Squares of Cycle Times of Raw Data by Film Shots

(Continued)

Operator	Shot	Sums of Cycle Times in Frames	Sums of Squares of Cycle Times in Frames	Number Cycles Per Shot
B3-17R	1	2,623	698,703	10
	2	2,356	621,106	9
	3	3,069	787,307	12
	4	1,596	426,676	6
	5	3,042	776,372	12
	6	3,365	881,147	13
	7	3,191	788,679	13
	8	3,292	843,288	13
B3-18Q	1	2,752	760,004	10
	2	3,093	882,079	11
	3	3,447	921,501	13
	4	2,711	739,037	10
	5	3,185	850,279	12
	6	2,902	772,092	11
	7	3,385	889,465	13
	8	3,039	773,597	12
B3-19N	1	1,812	674,272	5
	2	2,431	740,383	8
	3	3,602	1,201,602	11
	4	2,352	792,252	7
	5	2,601	767,315	9
	6	3,340	1,126,788	10
	7	3,047	1,043,811	9
	8	3,376	1,149,842	10

OPERATOR B2-10M

SHOT	1	2	3	4	5	6	7	8
T_i^2	8,340,544	4,223,025	5,851,561	7,573,504	11,992,369	13,697,401	—	14,615,329
n_i	9	8	8	9	12	12		12
$\frac{T_i^2}{n_i}$	926,727	527,878	731,445	841,500	999,364	1,141,450		1,217,944

$T^2 = \underline{445,252,201}$ $\quad N = \underline{70}$ $\quad \sum\sum X_{ij}^2 = \underline{6,562,083}$

MEANS $\quad \sum \frac{T_i^2}{n_i} - \frac{T^2}{N} = \underline{6,386,308} - \underline{6,360,746} = \underline{25,562}$

WITHIN: $\quad \sum\sum X_{ij}^2 - \sum \frac{T_i^2}{n_i} = \underline{6,562,083} - \underline{6,386,308} = \underline{175,775}$

TOTAL: $\quad \sum\sum X_{ij}^2 - \frac{T^2}{N} = \underline{6,562,083} - \underline{6,360,746} = \underline{201,337}$

	SUM OF SQUARES	D. F.	MEAN SQUARE	F RATIO
MEANS	25,562	6	4,260	$F = \frac{4,260}{2,790} = 1.527$
WITHIN	175,775	63	2,790	
TOTAL	201,337	69		

$F_{95}\ (6,63) = 2.3$

$F_{80}\ (6,63) = 1.5$

Figure 14. Sample Calculations and Formulae Used for the Analysis of Variance

Operator B2-10M

Class Interval Frames	Class Mark Frames	f	d	df	d^2f	d^3f	$\lvert X - \bar{X}\rvert f$
239.5-259.5	249.5	4	-4	-16	64	-256	57.1 f
259.5-279.5	269.5	17	-3	-51	153	-459	37.1 f
279.5-299.5	289.5	14	-2	-28	56	-112	17.1 f
299.5-319.5	309.5	12	-1	-12	12	- 12	2.9 f
319.5-339.5	329.5	10	0	0	0	0	22.9 f
339.5-359.5	349.5	5	1	5	5	5	42.9 f
359.5-379.5	369.5	3	2	6	12	24	62.9 f
379.5-399.5	389.5	0	3	0	0	0	--
399.5-419.5	409.5	3	4	12	48	192	102.9 f
419.5-439.5	429.5	1	5	5	25	125	122.9 f
Totals		69		-79	375	-493	2197.1
				$\bar{d}$	v_2	v_3	
1/N x Sums				-1.145	5.434	-7.144	

$\bar{d}^2 = 1.310 \qquad \bar{d}^3 = -1.500$

<u>Mean</u>

$\bar{X} = c\bar{d} + X_o = 20(-1.145) + 329.5 = 306.6$

<u>Variance and Standard Deviation</u>

$\mu_2 = v_2 - \bar{d}^2 = 5.434 - 1.310 = 4.124$

$\sigma_x = c\ \sqrt{\mu_2} = 40.62$

<u>Skewness</u>

$\mu_3 = v_3 - 3v_2\bar{d} + 2\bar{d}^3 = -7.144 - 3(5.434)(-1.145) + 2(-1.500)$

$= -7.144 + 18.666 - 3.000 = 8.522$

$\gamma_1 = \dfrac{\mu_3}{\sigma_u{}^3} = \dfrac{8.522}{8.376} = 1.02$

<u>Peakedness</u>

$a = \dfrac{\text{mean deviation}}{\text{standard deviation}} = \dfrac{\Sigma\lvert X_i - \bar{X}\rvert/N}{\sigma_x} = \dfrac{2197.1/69}{40.62} = 0.784$

Figure 15. Sample Calculations and Formulae for Determination of Moments

OPERATOR B2-10M

Upper Class Limit	t	Cumulative Area	Interval Area	Normal Freq.	Observed Freq.	$\frac{(f - F)^2}{F}$
$-\infty$						
			.1230	8.5	4	2.382
259.5	-1.16	.1230				
			.1284	8.9	17	7.371
279.5	-0.67	.2514				
			.1811	12.5	14	0.180
299.5	-0.17	.4325				
			.1930	13.3	12	0.127
319.5	0.32	.6255				
			.1655	11.4	10	0.172
339.5	0.81	.7910				
			.1122	7.7	5	0.947
359.5	1.30	.9032				
			.0968	6.7	7	0.013
$+\infty$						
			1.0000	69.0	69	11.192

$\bar{X} = 306.6$ $\sigma_x = 40.62$ Degrees of Freedom = 4

Probability Level 0.02

Figure 16. Sample Calculations for Fitting the Normal Curve to the Experimental Distributions.

OPERATOR B2-10M

Upper Class Limit	Log X	t	Cumulative Area	Interval Area	Log-Normal Freq.	Observed Freq.	$\frac{(f - F)^2}{F}$
$-\infty$							
				.1057	7.3	4	1.492
259.5	2.4141	-1.25	.1057				
				.1489	10.3	17	4.358
279.5	2.4464	-0.66	.2546				
				.1976	13.6	14	0.012
299.5	2.4764	-0.12	.4522				
				.1995	13.8	12	0.235
319.5	2.5045	0.39	.6517				
				.1562	10.8	10	0.059
339.5	2.5308	0.87	.8079				
				.0987	6.8	5	0.476
359.5	2.5557	1.32	.9066				
				.0934	6.4	7	0.056
$+\infty$							
				1.0000	69.0	69	6.688

$\overline{\text{Log } X} = 2.4830$ $S_{\text{Log } X} = 0.055$ Degrees of Freedom = 4

Probability Level 0.16

Figure 17. Sample Calculations for Fitting the Log Normal Curve to the Experimental Distributions.

OPERATOR B2-10M

Upper Class Limit	t	Cumulative Area	Interval Area	Pearson Type III	Observed Freq.	$\frac{(f - F)^2}{F}$
$-\infty$						
			.0902	6.2	4	0.781
259.5	-1.16	.0902				
			.1869	12.9	17	1.303
279.5	-0.67	.2771				
			.2204	15.2	14	0.095
299.5	-0.17	.4975				
			.1833	12.6	12	0.029
319.5	0.32	.6808				
			.1307	9.0	10	0.111
339.5	0.81	.8115				
			.0833	5.7	5	0.086
359.5	1.30	.8948				
			.1052	7.3	7	0.012
$+\infty$						
			1.0000	68.9	69	2.416

$\bar{X} = 306.6$ $\sigma_x = 40.62$ $\gamma_1 = 1.0$

Degrees of Freedom = 3

Probability Level 0.50

Figure 18. Sample Calculations for Fitting a Pearson Type III Curve to the Experimental Distributions.

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